Numerical Stability of NS and the Pressure correction term

Consider the NS equations for a transient flow along 1 direction.

Now, discretizing in time with Forward Euler and pressure implicitly, we have

Clearly, taking the divergence of this equation to ensure the velocity is zero at the next time step, we have

We are already familiar with the numerical stability requirements of the diffusion and advection terms:

Note that if the pressure equation is not solved exactly, there is an additional restriction on the time step size related to the pressure correction term. Expressing the numerical stability requirement of the pressure term is the central effort of this report.

I believe that all of these numerical stability criterion are related, in one way or another, to the divergence of the quantity in question. That is, divergence due to diffusion, divergence due to advection and divergence due to lack of solving exactly the pressure poisson equation.

Looking at the correction term,

we can see that taking the divergence of this yields

Which is equivalent to solving, exactly, the same equation with an error added:

or

Where

or

Look at Von Neumann Analysis to proceed with this example.

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Try following procedure in

http://en.wikipedia.org/wiki/Von\_Neumann\_stability\_analysis

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Now let's discretize